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Characteristics of Low-Frequency Acoustic Wave Propagation in Ice-Covered Shallow Water Environment

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Abstract: Mastering the sound propagation law of low-frequency signals in the Arctic is a major frontier basic research demand to improve the level of detection, communication, and navigation technology. It is of practical significance for long-distance sound propagation and underwater target detection in the Arctic Ocean. Therefore, how to establish an effective model to study the characteristics of the acoustic field in the Arctic area has always been a hot topic in polar acoustic research. Aimed at solving this problem, a mathematical polar acoustic field model with an elastic seafloor is developed based on a range-dependent elastic parabolic equation theory. Moreover, this method is applied to study the characteristics of polar sound propagation for the first attempt. The validity and effectiveness of the method and model are verified by the elastic normal mode method. Simultaneously, the propagation characteristics of low-frequency signals are studied in a polar sound field from three aspects, which are seafloor parameters, sea depth, and ice thickness. The results show that the elastic parabolic equation method can be well utilized to the Arctic low-frequency acoustic field. The analysis of the influence factors of the polar sound field reveals the laws of sound transmission loss of low-frequency signals, which is of great significance to provide information prediction for underwater submarine target detection and target recognition.

Keywords: low frequency; polar shallow water environment; elastic parabolic equation

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1. Introduction

In recent years, global warming has led to a decrease in the area of Arctic ice, which has caused the world's marine powers to pay unprecedented attention to the Arctic [1]. In order to meet the needs of energy and minerals, fishery resources and the ecological environment, tourism and transportation, climate change and national defense security in the Arctic, countries think highly of the research on the sound propagation characteristics of low-frequency signals in the Arctic. Therefore, scholars have studied the models and modeling methods of the Arctic region. Burke [2] firstly considered the ice interface with ridge distribution as an infinite semi elliptical cylinder, which was randomly distributed on a rigid or free interface and established the Burke–Twersky (BT) model to describe the sound field under the ice. Diachok [3] analyzed the relationship between the reflection loss and the size, number, and distribution density of ice ridges at different frequencies on the basis of the BT model, and further studied the propagation loss under the ice using the ray theory. Wolf [4] proposed an improved BT model, and the solution is consistent with the experimental results from the polar region in the low-frequency range. In order to study the polar sound field with the more realistic polar model, the perturbation theory has been used to calculate the scattering coefficients of elastic modes [5]. Kuperman [6,7] used the wave number integration method and the perturbation theory to derive the low-frequency reflection loss at the interface of the elastic medium, and he calculated the sound propagation loss under the Arctic ice. Jon [8] applied the elastic parabolic equation to polar acoustic for the first time and verified the accuracy of the method. Scott [9] developed a full-field perturbation method to estimate the narrow-band long-range reflection caused by the roughness of the ice water interface. Anatoliy [10] compared the reverberation of the fluid model ice with that of elastic model ice for an upward refracted acoustic velocity profile, and he further considered the reverberation of the elastic seafloor with a nearly fluid mud layer. Previous research mainly focused on ice modeling and analysis methods to study whether they are suitable for polar acoustics.

Different from research studies above, this paper further considers the influence of some specific factors on the sound propagation characteristics of polar sound field from the perspectives of the elastic seafloor and the low-frequency signal. Section 2 details the elastic parabolic equation method and the polar acoustic numerical model of ice–water–seafloor. The ice layer and the seafloor are regarded as elastic media in this model [11,12]. The accuracy of the polar sound field model is verified in Section 3. Section 4 focuses on the sound propagation characteristics of the traditional polar sound field model from three aspects: ice thickness, seawater depth, and seafloor medium parameters, and it reveals the propagation law of low-frequency signals in a polar shallow water sound field.

2. Elastic Model

Based on the elastic parabolic equation, the polar sound field model proposed in the paper is shown in Figure 1.

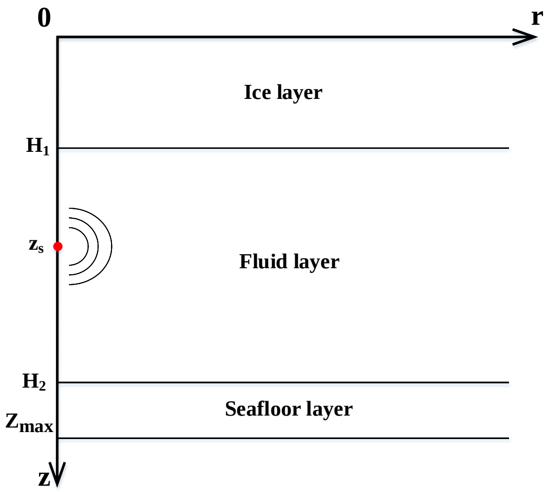


Figure 1. Schematic diagram of polar sound field model.

It is assumed that the ice layer is the elastic layer above the surface of a homogeneous fluid layer and the sound field has cylindrical symmetry in the near range of the sound source. The impulse function is used to replace the time-harmonic point source for underwater sound propagation problems. The z-axis is vertically downward, which represents the depth of the ocean, and range r represents the horizontal distance from the sound source. The ice–air interface is at $z = 0$, the top of the ice layer is located at $z = H_1$, where it is also the ice–water interface. The seafloor layer is at z = H_2. The point sound source is in the fluid layer at range $r = 0$ and depth $z = z_s$.

2.1. Elastic Parabolic Equation Method with Ice Covers

The parabolic equation can deal with the large-angle problem by approximating the far field and ignoring the inward propagating wave. The accuracy and stability of the solution are greatly improved by the split step method, and the improved self-starter can deal with various boundary conditions effectively.

Based on the relationship between stress and displacement, the elastic parabolic equation is derived from the equation of motion for elastic medium, which are written as the (u_r,w) formulation of elasticity, for $u_r = \partial u/\partial r$ [13],

$$(\lambda + 2\mu)\frac{\partial^2 u_r}{\partial r^2} + \frac{\partial}{\partial z}\left(\mu\frac{\partial u_r}{\partial z}\right) + \rho\omega^2 u_r + (\lambda + \mu)\frac{\partial^3 w}{\partial r^2 \partial z} + \frac{\partial \mu}{\partial z}\frac{\partial^2 w}{\partial r^2} = 0 \tag{1}$$

$$\mu\frac{\partial^2 w}{\partial r^2} + \frac{\partial}{\partial z}\left((\lambda + 2\mu)\frac{\partial w}{\partial z}\right) + \rho\omega^2 w + (\lambda + \mu)\frac{\partial u_r}{\partial z} + \frac{\partial \lambda}{\partial z}u_r = 0 \tag{2}$$

where u_r is the derivative of the horizontal displacement with respect to range and w is the vertical displacement. ω is a time-harmonic point source of angular frequency. λ and μ are Lame elastic parameters and defined as,

$$\lambda = \rho\left(c_p^2 - 2c_s^2\right) \tag{3}$$

$$\mu = \rho c_s^2 \tag{4}$$

where c_p is a compressional wave, c_s is a shear wave, and ρ is medium density.

Through the derivation and transformation of Equations (1) and (2) in the form of a matrix operator, the standard form of a elastic parabolic equation is written. It is transformed into the form of inward and outward propagation, ignoring the outward propagation component. The solution of the equation is given by the differential:

$$\left.\begin{pmatrix} u_r \\ w \end{pmatrix}\right|_{r+\Delta r} = e^{ik_0\Delta r\sqrt{I+X}}\left.\begin{pmatrix} u_r \\ w \end{pmatrix}\right|_{r} \tag{5}$$

with

$$X = {k_0}^{-2}\left(L^{-1}M - {k_0}^2 I\right) \tag{6}$$

where Δr is a step in the range, I is the identity matrix, k_0 is the reference wavenumber, and L and M are a matrix containing medium parameters and depth, respectively [14].

By applying the split-step Pade approximation for Equation (3), it is obtained that

$$\left.\begin{pmatrix} u_r \\ w \end{pmatrix}\right|_{r+\Delta r} = e^{ik_0\Delta r}\prod_{j=1}^{n}\frac{1+\alpha_{j,n}}{1+\beta_{j.n}}\left.\begin{pmatrix} u_r \\ w \end{pmatrix}\right|_{r} \tag{7}$$

where $\alpha_{j,n}$ and $\beta_{j,n}$ should meet the requirements of stability, convergence, and accuracy.

The discretization of depth operator X is based on the Galerkin method [15]. After discretization, a large sparse banded matrix can be obtained, which can be solved by the block pursuit method based on Gaussian principal elimination. For the storage of the matrix, according to the characteristics of only six elements in each row, six groups of data can be used to store the coefficient matrix. Applying the initial condition at $r = 0$ by using the self-starter [16] and the sparse matrix, the solution can be obtained by solving step by step.

After obtaining the whole sound field, the sound propagation characteristics are expressed by the sound pressure transmission loss:

$$TL = -20 * \log\frac{P}{P_0} \tag{8}$$

where P represents the sound pressure amplitude obtained by simulation, and P_0 represents the sound pressure at the sound source.

2.2. Boundary Condition

In this model, the ice layer and seafloor are regarded as an elastic medium. When the sound wave enters the elastic medium, it will produce two kinds of waves: compressional wave and shear wave [17]. In certain conditions, three complex interface waves may be excited at the interface [18], such as the Rayleigh wave (air–elastic interface) [19] and Scholte wave (fluid–elastic interface) [20]. Therefore, the boundary conditions of the elastic parabolic equation are considered from the above two aspects.

Generally, the air–elastic interface is regarded as the pressure release interface, which meets the zero traction condition, it is expressed as

$$\lambda_1 \Delta_1 = 0 \tag{9}$$

$$\lambda_2 u_{r2} + (\lambda_2 + 2\mu_2)\frac{\partial w_2}{\partial z} = 0 \tag{10}$$

$$\frac{\partial}{\partial z}(\lambda_2 u_{r2}) + \frac{\partial}{\partial z}(\lambda_2 + 2\mu_2)\frac{\partial w_2}{\partial z} + \rho_2 \omega^2 w_2 = 0 \tag{11}$$

where subscript 1 and subscript 2 represent the air layer and the elastic layer, respectively.

The boundary conditions of the fluid–elastic interface should satisfy that the vertical stress is continuous, the displacement is continuous, and the tangential stress is zero [21], which is written as

$$\frac{\partial}{\partial z}(\lambda_2 \Delta_2) + \rho_2 \omega^2 w_2 = 0 \tag{12}$$

$$\lambda_2 \Delta_2 = \lambda_3 \Delta_3 + 2\mu_3 \frac{\partial w_3}{\partial z} \tag{13}$$

$$\frac{\partial}{\partial z}(\lambda_2 u_{r2}) + \frac{\partial}{\partial z}(\lambda_2 + 2\mu_2)\frac{\partial w_2}{\partial z} + \rho_2 \omega^2 w_2 = 0 \tag{14}$$

where the subscript 3 represents the fluid layer.

In the discrete process of parabolic equation, the interface conditions may be handled by introducing artificial grid points; we placed the interface midway between real and artificial grid points in both layers [22]. The interface condition is discretized by the central difference method and substituted into the discrete parabolic equation. The artificial quantity can be eliminated at the interface.

3. Model Validation

In this part, the elastic normal mode [23] is utilized to demonstrate the correctness of the ice–fluid–seabed model. This method is different from other methods such as ray theory, which is advantageous for studying the sound propagation characteristics of low-frequency sound signals.

Medium parameters about the ice-cover shallow water model proposed in this paper are shown in Table 1. For the model of elastic normal mode and elastic parabolic equation with the same parameters, the sound field characteristics are analyzed respectively. When the frequency of the sound source is 50 Hz, the comparison between the transmission loss of the elastic normal mode model and elastic parabolic equation model are shown in Figure 2.

It can be seen from Figure 2 that two curves signifying the solutions of elastic parabolic solution and elastic normal mode coincide with each other. Moreover, under the same medium parameters, the solution is in agreement with that of Collis [8]. Through these verifications, this model is correct, and the method of an elastic parabolic equation can be used in a polar sound field model reasonably.

Table 1. Model physical parameters.

Layer Type	Parameter	Value
Ice layer	Ice thickness (m)	3
	Ice density (g/cm^3)	0.9
	Compressional speed ice (m/s)	3500
	Shear speed ice (m/s)	1800
	Compressional attenuation ice (dB/λ)	0.3
	Shear attenuation ice (dB/λ)	1.0
Fluid layer	Fluid depth (m)	100
	Fluid density (g/cm^3)	1.0
	Compressional speed fluid (m/s)	1482
Seafloor layer	Seafloor density (g/cm^3)	2.2
	Compressional attenuation seafloor (dB/λ)	0.76
	Shear attenuation seafloor (dB/λ)	1.05

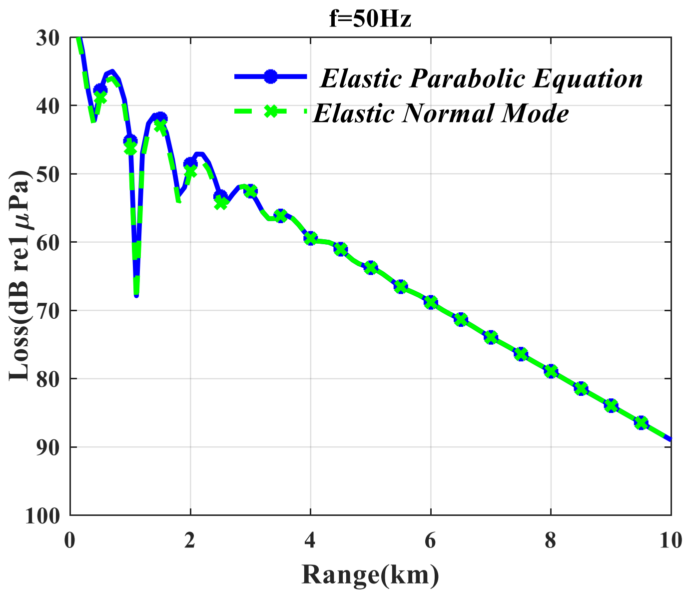


Figure 2. Verification of two different methods for the proposed model.

4. Parameter Analysis

On the basis of the proper polar sound field model, exploration is made about the sound propagation characteristics of low-frequency signals in the polar environment. This section considers the effect of acoustic propagation loss on low-frequency signals in a polar shallow water acoustic field environment under different parameters.

4.1. The Influence of Ice Thickness

The acoustic parameters of an ice layer include ice layer density, thickness, and compressional wave and shear wave velocities. Considering that ice density has little effect on sound propagation, especially on long-distance sound propagation, it is neglected. The influence of ice thickness on sound field distribution is mostly considered here. Since the ice thickness is continuous fluctuation, it is necessary to study the influence of different ice thicknesses on sound propagation. The frequency of the sound source is 50 Hz, the depth of the sound source is 30 m, the depth of the sound receiver is 30 m, the seawater depth is 100 m, and the thickness of the ice layers are 5 m, 10 m, 15 m, and 20 m respectively. The results of numerical analysis are shown in Figures 3 and 4.

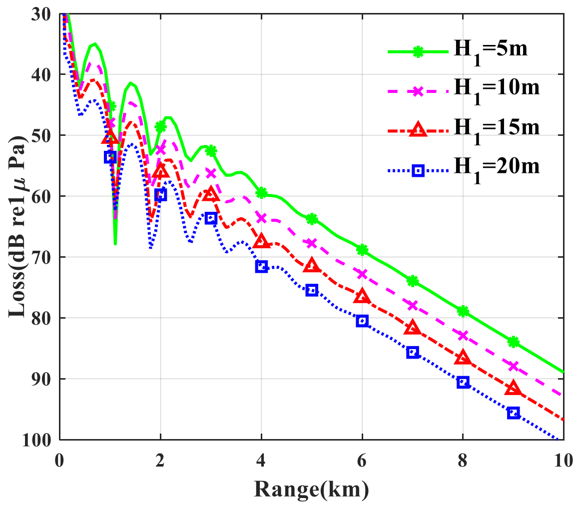
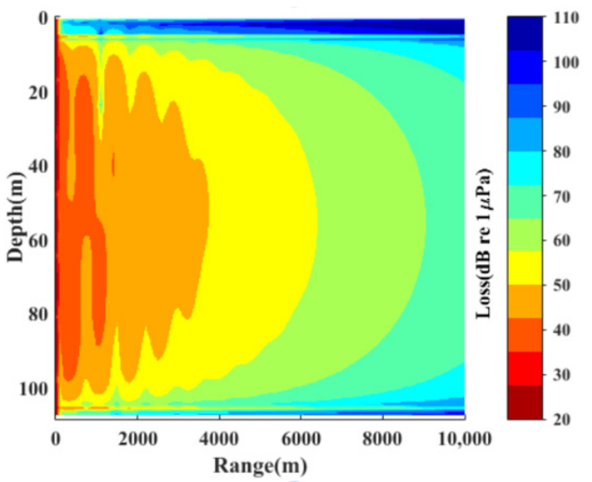
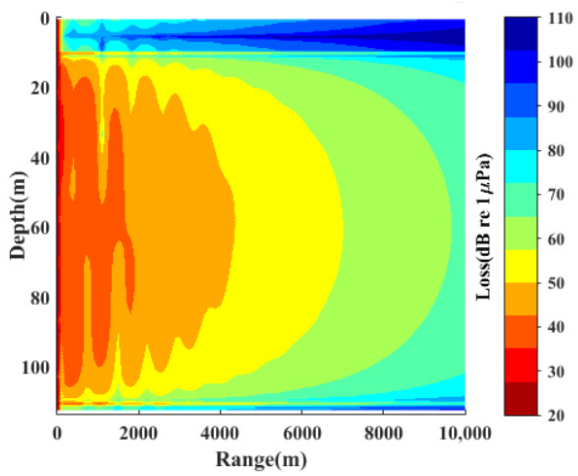


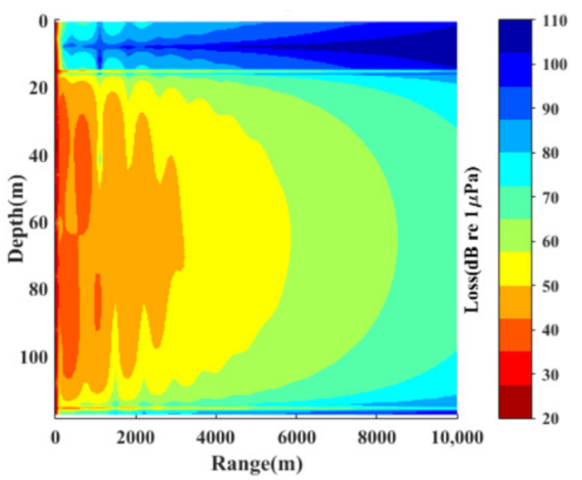
Figure 3. Curves of propagation loss at different ice thicknesses.



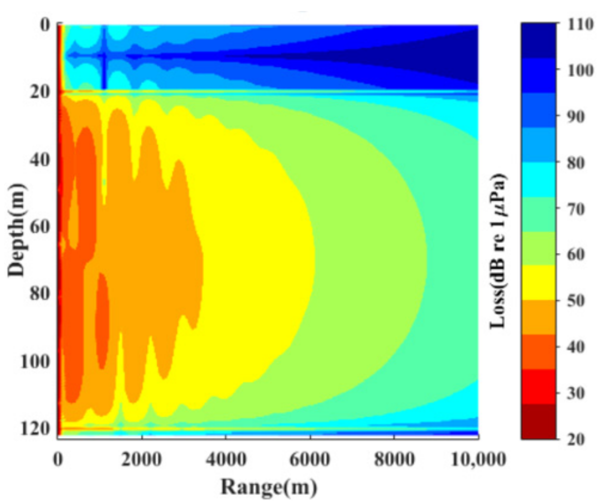
(**a**) Ice layer $H_1 = 5$ m



(**b**) Ice layer $H_1 = 10$ m



(**c**) Ice layer $H_1 = 15$ m



(**d**) Ice layer $H_1 = 20$ m

Figure 4. Contour plots (**a**–**d**) of propagation loss at different ice thicknesses.

Figures 3 and 4 show the transmission loss curves and contour plots at different ice thicknesses, respectively. The results show that when the distance between the sound source location and the ice layer is constant, the propagation loss increases with the increase of ice thickness. This is primarily because the thicker the ice layer is, the more the energy of compression wave in seawater penetrates into the seafloor, resulting in greater propagation loss.

4.2. The Influence of Seafloor Parameters

The seabed environment is complex and changeable, so the influence of the seafloor medium parameters is considered on the sound propagation characteristics. The characteristics of seafloor parameters are chiefly manifested in compressional wave velocity and shear wave velocity. The sound source frequency is 50 Hz, the sound source and the sound receiver are at the depth of 30 m and 50 m, respectively, the thickness of the ice is 5 m, the following groups are selected for the sound velocity of a compressional wave and shear wave: one is that the compressional wave velocity and shear wave velocity are 3800 m/s and 1800 m/s, respectively, and the other is that the compressional wave velocity and shear wave velocity are 2400 m/s and 1200 m/s, respectively.

It can be seen from Figure 5 that the propagation loss of the first seafloor is lower than that of the other seafloor, and the attenuation laws of the two lines are different. The sound field environment is different under different medium parameters. Under the first seabed (compressional wave velocity and shear wave velocity are 3800 m/s and 1800 m/s), a Scholte surface wave is excited, and there is a waveguide normal wave in the water. Under the other seabed (compressional wave velocity and shear wave velocity are 2400 m/s and 1200 m/s), no Scholte surface wave is excited, and there is only an attenuated normal wave in the water. Obviously, different dielectric parameters can lead to the change of sound field, which leads to different attenuation modes. With the decrease of shear wave velocity, the seabed medium behaves as the fluid medium, and the ability of the absorbing surface wave energy decreases. When the shear wave velocity is 0, it is equivalent to the infinite deep seafloor, and the propagation loss reaches the maximum.

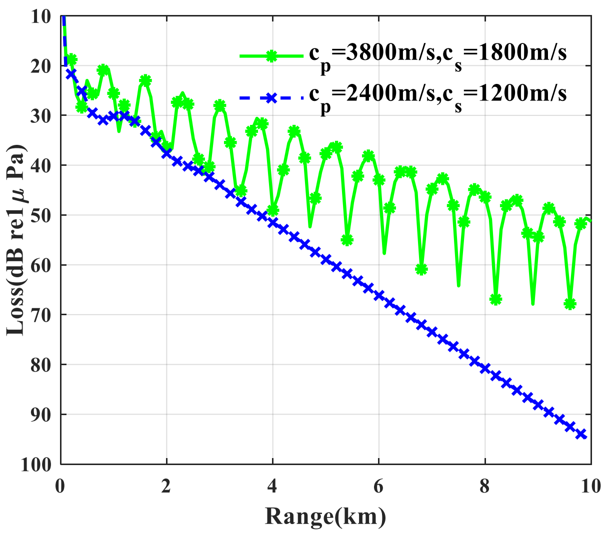
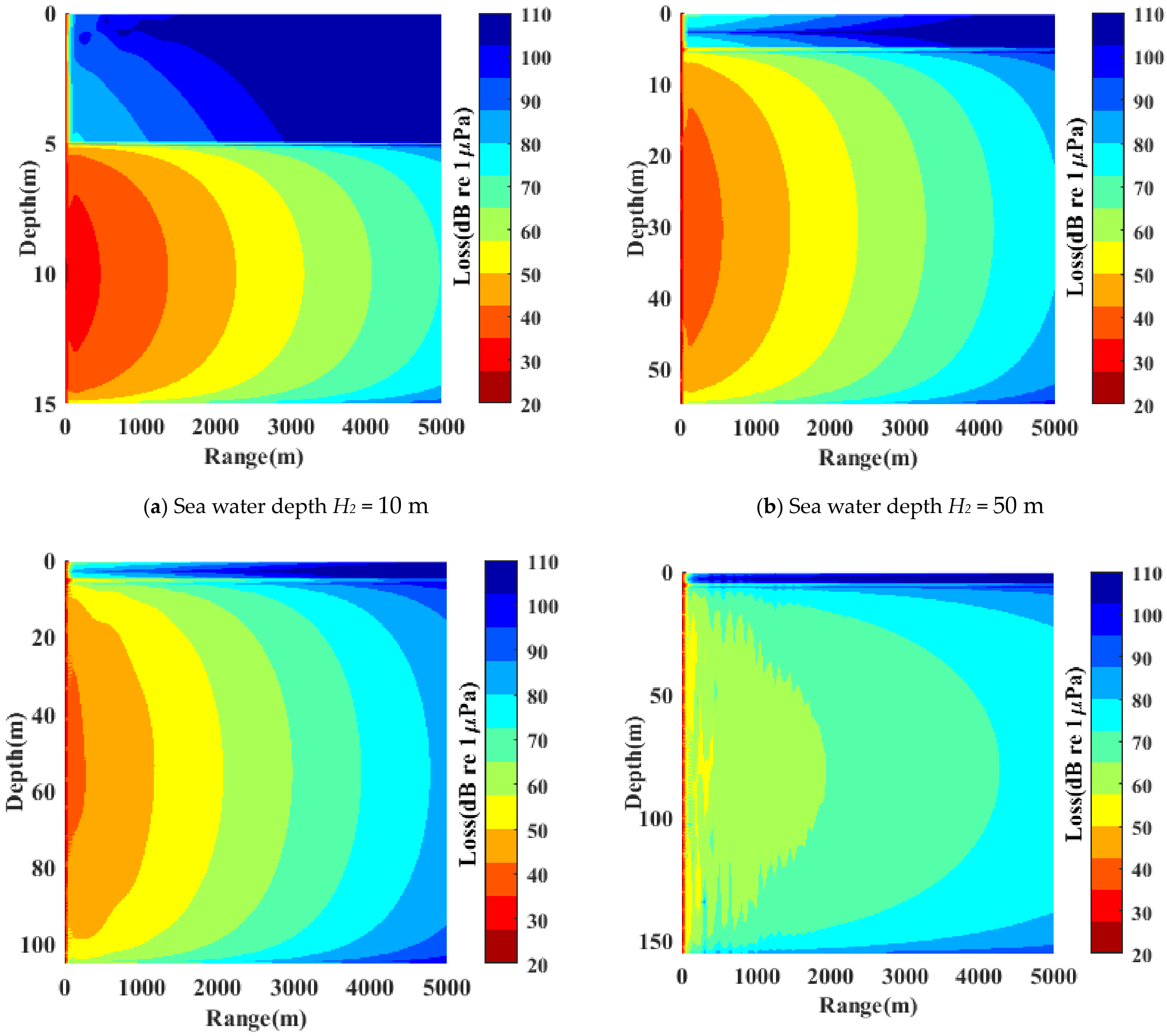


Figure 5. Curves of propagation loss in different seafloor parameters.

4.3. The Influence of Sea Water Depth

The parameters of the sea water depth sound field are the same as before, only changing the sea water depth h, and the sound source is in the middle of the sea water. Figure 6 shows the propagation loss and contour plots when the sea water depth is 10 m, 50 m, 100 m, and 150 m, respectively.



(**a**) Sea water depth H_2 = 10 m (**b**) Sea water depth H_2 = 50 m

(**c**) Sea water depth H_2 = 100 m (**d**) Sea water depth H_2 = 150 m

Figure 6. Contour plots (**a**–**d**) of propagation loss at different underwater depths.

It can be seen from Figure 6 that when the water depth is relatively shallow (10 m), a large part of the compressional wave energy in the seawater penetrates into the seafloor and ice layer at a short distance, and only a small part is converted into Scholte surface wave propagation. Other conditions remain unchanged. With the increase of seawater depth, the relative distance between the sound source and the ice layer and the seafloor increases, and the number of collisions between the sound line and the upper interface decreases, which is conducive to the long-distance transmission of sound.

5. Conclusions

This paper firstly applies the elastic parabolic equation to study the acoustic propagation characteristics and influencing factors of low-frequency signals in a polar shallow water environment. Combining the elastic parabolic equation method and boundary conditions, a numerical model of a polar acoustic field is established. The ice layer and sea bottom of the model is regarded as an elastic medium, and its effect is considered. The validity and accuracy of the elastic parabolic equation method are verified by the elastic normal mode method. The results show that the method can be well applied to the study of low-frequency signals in polar shallow water. At the same time, the influence factors of low-frequency sound propagation in a polar environment are analyzed, which reveals the influence of several key parameters on the propagation characteristics of sound pressure in water. With the decrease of distance between the sound source and ice layer, the value of the propagation loss is larger. A thicker ice layer and smaller compressional wave velocity of the seafloor also cause larger propagation loss. In these cases, it is not conducive to the long-distance transmission of acoustic signals. This research has important value in the field of underwater acoustic scientific research and engineering applications. At the same time, understanding the Arctic environmental sound field can be helpful to improve the environmental adaptability of ship sonar equipment and meet the research needs of detection, communication, and navigation technology. The polar sound field environment simulated in this paper is parallel to the seabed. More complex marine environments such as tilt are not discussed, and the characteristics of sound vector parameters need to be further studied.

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